

Intensity Transformations and Spatial Filtering

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What will We Learn today?

- Histogram of an image
- Normalized histogram
- Histogram equalization

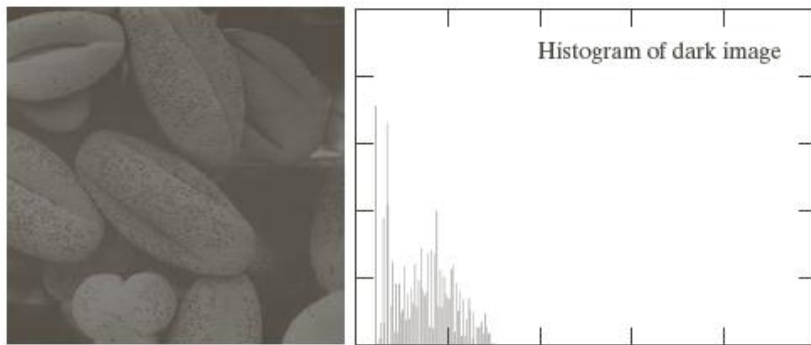
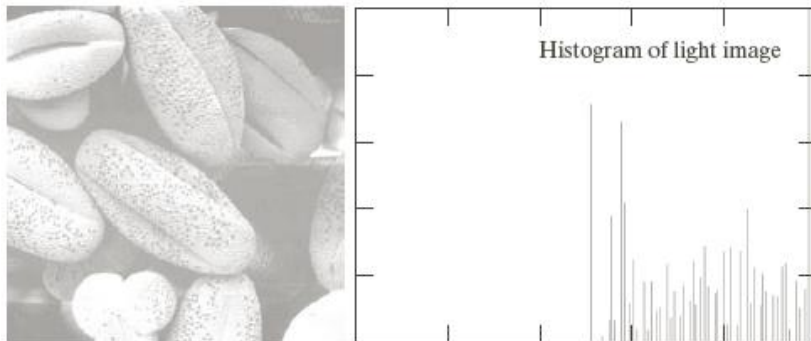
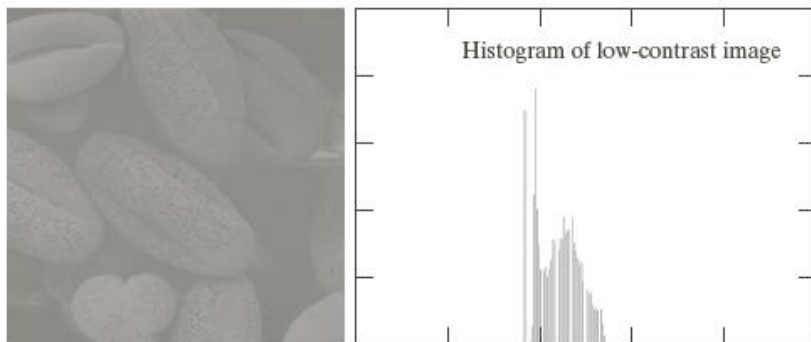
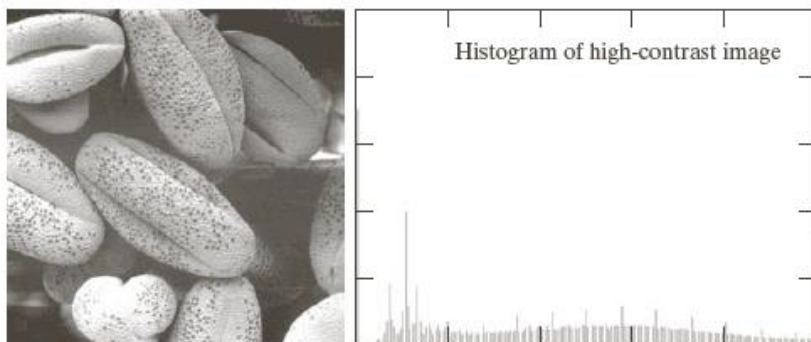


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

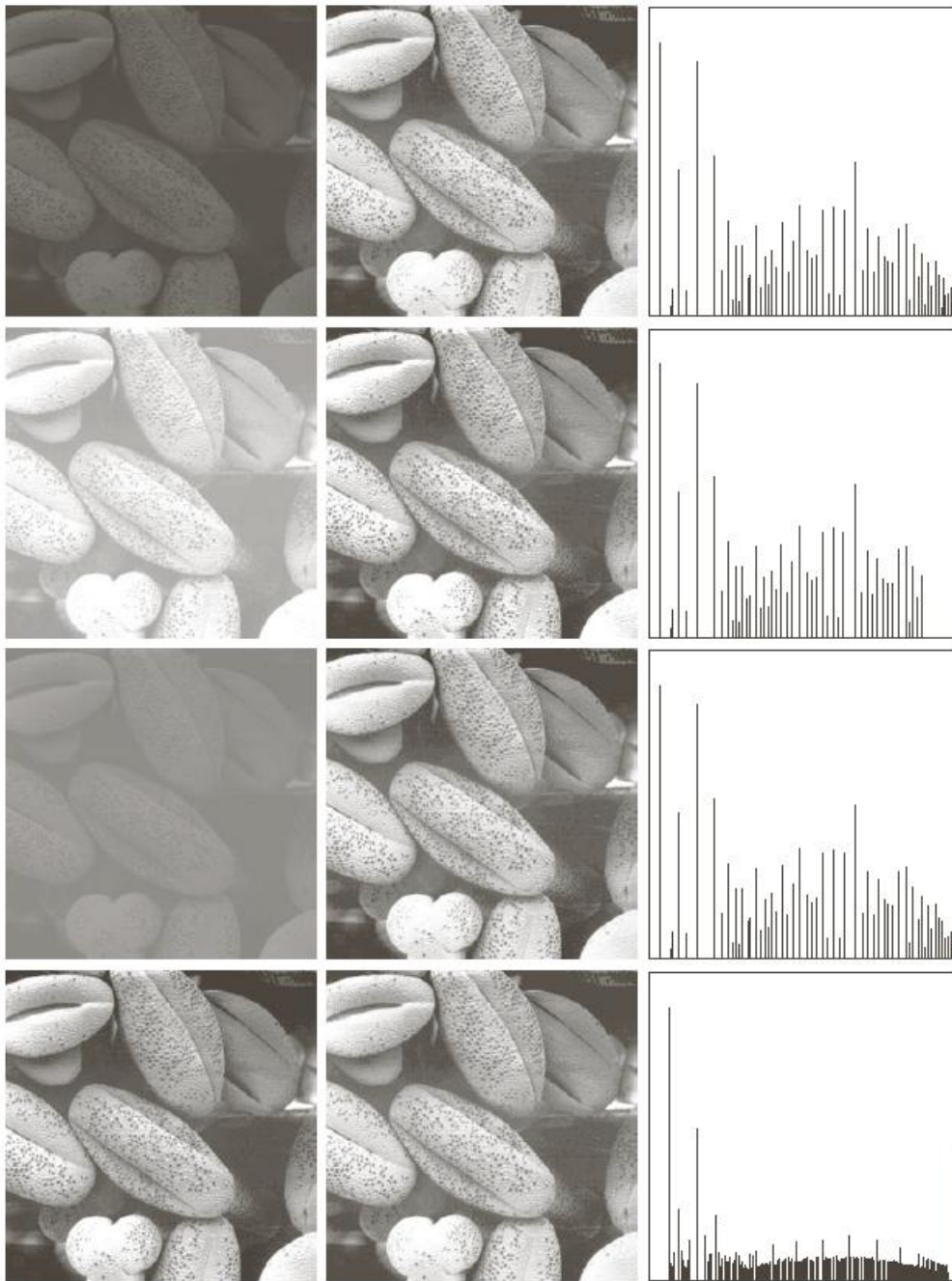
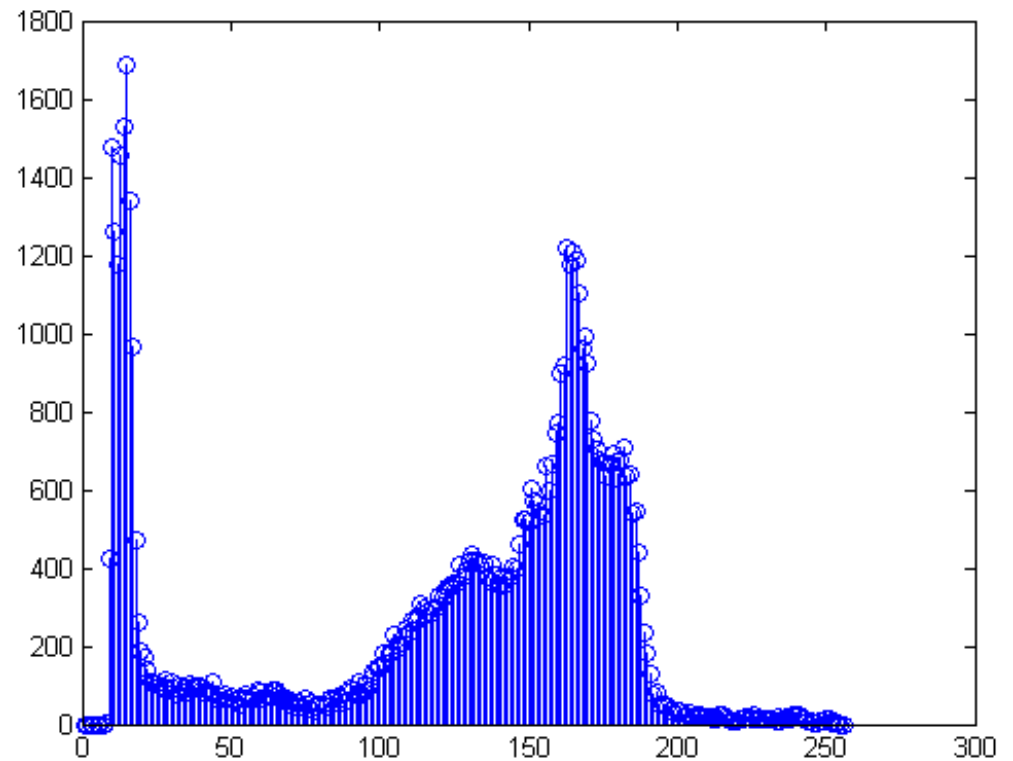


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

What is a histogram?



Cameraman image



Histogram

What is a histogram?

Image (3-bit)

0	2	5	0
2	4	7	5
3	5	6	4
0	3	4	0

Example of a 3-bit image(gray level in the range 0-7)

$$h(r_k) = \begin{bmatrix} 4 \\ 0 \\ 2 \\ 2 \\ 3 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

Corresponding histogram

$$p(r_k) = \begin{bmatrix} \frac{4}{16} \\ \frac{0}{16} \\ \frac{2}{16} \\ \frac{2}{16} \\ \frac{3}{16} \\ \frac{3}{16} \\ \frac{1}{16} \\ \frac{1}{16} \end{bmatrix}$$

Normalized histogram

Histogram Processing

Histogram $h(r_k) = n_k$

r_k is the k^{th} intensity value

n_k is the number of pixels in the image with intensity r_k

Normalized histogram $p(r_k) = \frac{n_k}{MN}$

n_k : the number of pixels in the image of
size $M \times N$ with intensity r_k

Histogram Equalization

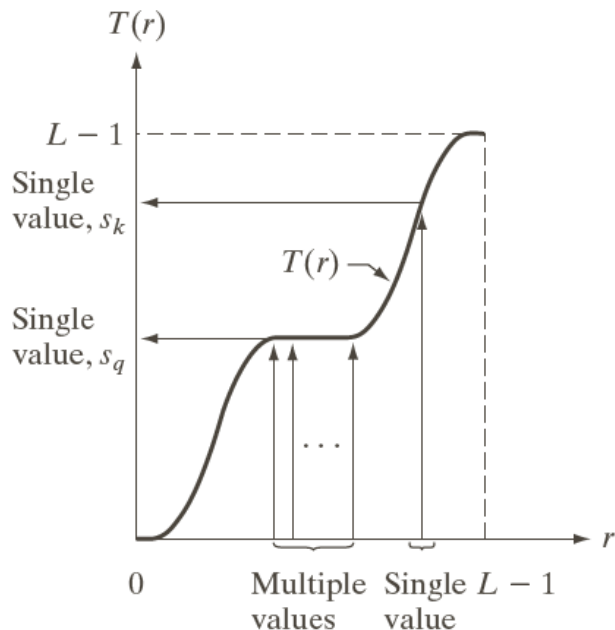
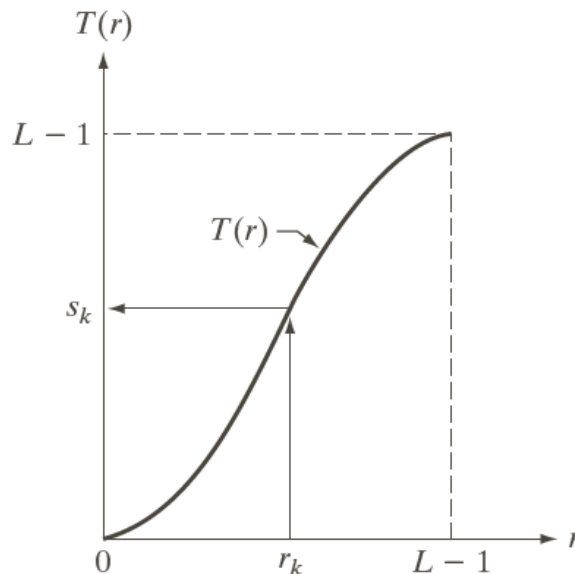
The intensity levels in an image may be viewed as random variables in the interval $[0, L-1]$.

Let $p_r(r)$ and $p_s(s)$ denote the probability density function (PDF) of random variables r and s .

Histogram Equalization

$$s = T(r) \quad 0 \leq r \leq L-1$$

- a. $T(r)$ is a strictly monotonically increasing function in the interval $0 \leq r \leq L-1$;
- b. $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$.



a b

FIGURE 3.17

(a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Equalization

$$s = T(r) \quad 0 \leq r \leq L-1$$

- a.* $T(r)$ is a strictly monotonically increasing function in the interval $0 \leq r \leq L-1$;
- b.* $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$.

$T(r)$ is continuous and differentiable.

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

Histogram Equalization

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

Discrete values:

$$s_k = T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j)$$

$$= (L - 1) \sum_{j=0}^k \frac{n_j}{MN} = \frac{L - 1}{MN} \sum_{j=0}^k n_j \quad k=0,1,\dots, L-1$$

Example: Histogram Equalization

Suppose that a 3-bit image ($L=8$) of size 64×64 pixels ($MN = 4096$) has the intensity distribution shown in following table. Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k .

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Discrete values:

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Example: Histogram Equalization

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$r_7 = 7$	81	0.02

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j)$$

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7 \times 0.19 = 1.33 \rightarrow 1$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \rightarrow 3$$

$$s_2 = 4.55 \rightarrow 5$$

$$s_3 = 5.67 \rightarrow 6$$

$$s_4 = 6.23 \rightarrow 6$$

$$s_5 = 6.65 \rightarrow 7$$

$$s_6 = 6.86 \rightarrow 7$$

$$s_7 = 7.00 \rightarrow 7$$

Example: Histogram Equalization

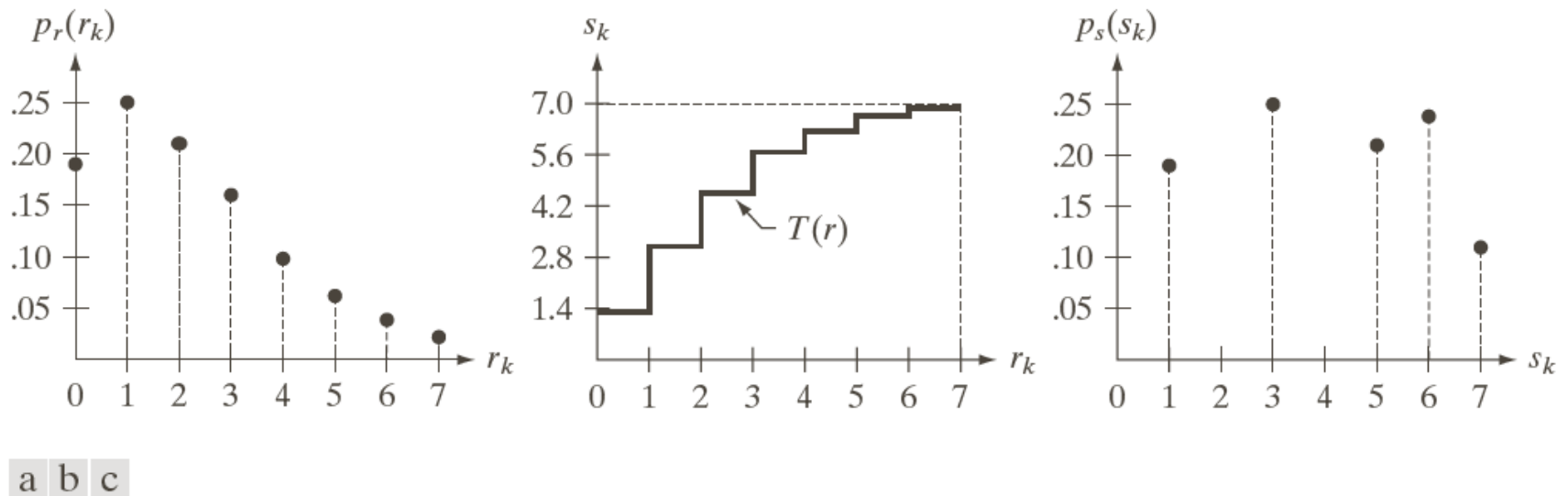


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.