Image Restoration & Reconstruction

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Spatial Filtering: Adaptive Filters (1)

Adaptive filters

The behavior changes based on statistical characteristics of the image inside the filter region defined by the mxn rectangular window.

The performance is superior to that of the filters discussed

Adaptive Filters:
Adaptive, Local Noise Reduction Filters (1)

S_{xy}: local region

The response of the filter at the center point (x,y) of S_{xy} is based on four quantities:

(a) $g(x, y)$, the value of the noisy image at (x, y);

(b) σ_η^2, the variance of the noise corrupting $f(x, y)$ to form $g(x, y)$;

(c) m_L, the local mean of the pixels in S_{xy};

(d) σ_L^2, the local variance of the pixels in S_{xy}.

Adaptive Filters:
Adaptive, Local Noise Reduction Filters (2)

The behavior of the filter:

(a) if σ_η^2 is zero, the filter should return simply the value of $g(x, y)$.

(b) if the local variance is high relative to σ_η^2, the filter should return a value close to $g(x, y)$;

(c) if the two variances are equal, the filter returns the arithmetic mean value of the pixels in S_{xy}.

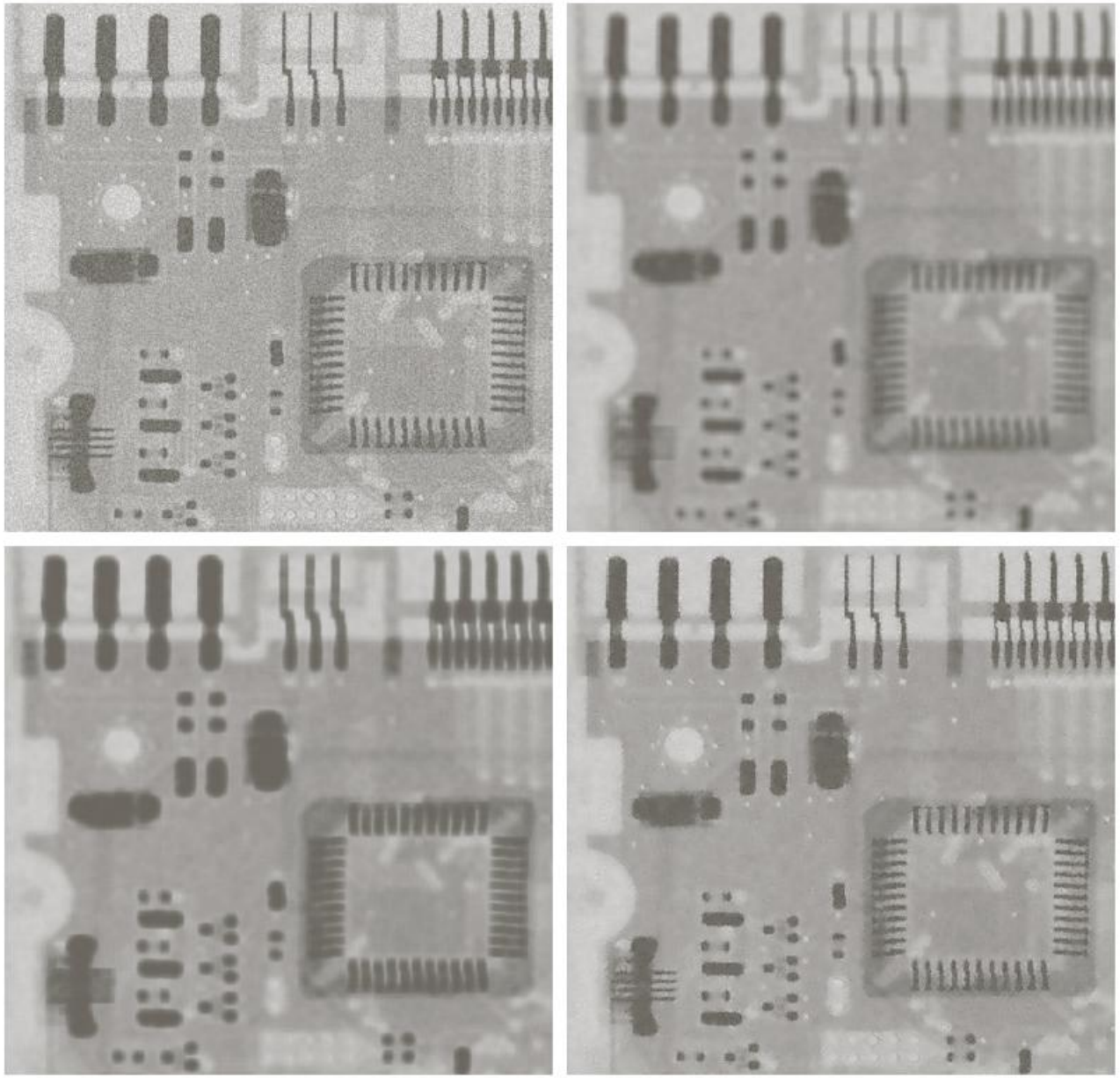
Adaptive Filters:
Adaptive, Local Noise Reduction Filters (3)

An adaptive expression for obtaining $f(x,y)$ based on the assumptions:

$$f(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

a b
c d

FIGURE 5.13
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.
(b) Result of arithmetic mean filtering.
(c) Result of geometric mean filtering.
(d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



Adaptive Filters:
Adaptive Median Filters (1)

The notation:

$z_{\min}$ = minimum intensity value in S_{xy}

$z_{\max}$ = maximum intensity value in S_{xy}

z_{med} = median intensity value in S_{xy}

z_{xy} = intensity value at coordinates (x, y)

$S_{\max}$ = maximum allowed size of S_{xy}

Adaptive Filters:
Adaptive Median Filters (2)

The adaptive median-filtering works in two stages:

Stage A:

$A1 = z_{med} - z_{min}$; $A2 = z_{med} - z_{max}$

if A1>0 and A2<0, go to stage B

Else increase the window size

if window size $\leq S_{max}$, repeat stage A; Else output z_{med}

Stage B:

$B1 = z_{xy} - z_{min}$; $B2 = z_{xy} - z_{max}$

if B1>0 and B2<0, output z_{xy}; Else output z_{med}

Adaptive Filters:
Adaptive Median Filters (2)

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Stage A:

$A1 = z_{med} - z_{min}$; $A2 = z_{med} - z_{max}$

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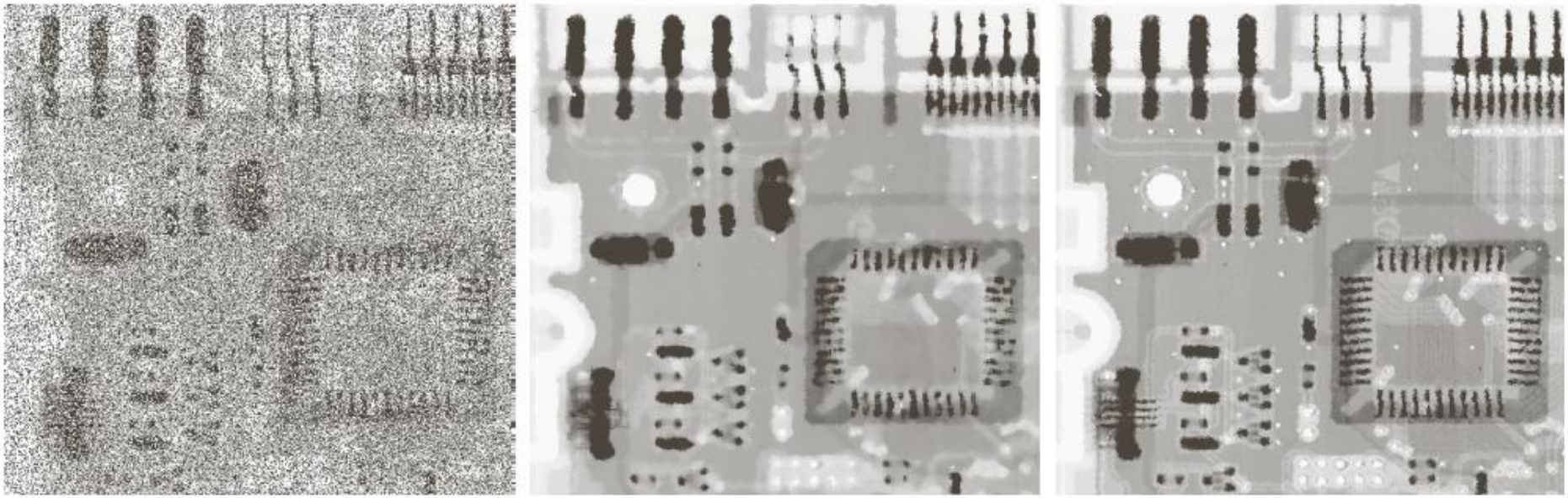
if window size $\leq S_{max}$, repeat stage A; Else output z_{med}

Stage B:

$B1 = z_{xy} - z_{min}$; $B2 = z_{xy} - z_{max}$

if B1>0 and B2<0, output z_{xy}; Else output z_{med}

Example:
Adaptive Median Filters



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

Minimum Mean Square Error (Wiener) Filtering

- **N. Wiener (1942)**
- **Objective**

  Find an estimate of the uncorrupted image such that the mean square error between them is minimized

$$e^2 = E\left\{(f - \hat{f})^2\right\} \qquad (5\text{-}80)$$

- ✓ where $E\{.\}$ i is the expected value of the argument.
- ✓ Assuming: noise and the image to be uncorrelated.

Minimum Mean Square Error (Wiener) Filtering

- **Objective**

  Find an estimate of the uncorrupted image such that the mean square error between them is minimized

$$e^2 = E\left\{(f - \hat{f})^2\right\} \qquad (5\text{-}80)$$

- ✓ Assuming: that the image or the noise has zero mean,
- ✓ Assuming: that the intensity levels in the estimate are a linear function of the levels in the degraded image.

Minimum Mean Square Error (Wiener) Filtering

on these assumptions, the minimum of the error function in Eq. (5-80) is given in the frequency domain by the expression

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$= \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v) \qquad (5\text{-}81)$$

$$= \left[\frac{1}{H(u,v)}\frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

Minimum Mean Square Error (Wiener) Filtering

The terms in Eq. (5-81) are as follows:

1. $\hat{F}(u,v)$ = Fourier transform of the estimate of the undegraded image.
2. $G(u,v)$ = Fourier transform of the degraded image.
3. $H(u,v)$ = degradation transfer function (Fourier transform of the spatial degradation).
4. $H^*(u,v)$ = complex conjugate of $H(u,v)$.
5. $|H(u,v)|^2 = H^*(u,v)H(u,v)$.
6. $S_\eta(u,v) = |N(u,v)|^2$ = power spectrum of the noise [see Eq. (4-89)]†
7. $S_f(u,v) = |F(u,v)|^2$ = power spectrum of the undegraded image.

Minimum Mean Square Error (Wiener) Filtering

- where we used the fact that the product of a complex quantity with its conjugate is equal to the magnitude of the complex quantity squared.

- This result is known as the *Wiener filter*,

- The filter, which consists of the terms inside the brackets, also is commonly referred to as the *minimum mean square error filter* or the *least square error filter*.

Minimum Mean Square Error (Wiener) Filtering

When dealing with white noise, the spectrum is a constant, which simplifies things considerably.

However, the power spectrum of the undegraded image seldom is known.

An approach frequently used when these quantities are not known, or cannot be estimated, is to approximate Eq. (5-81) by the expression,

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v) \qquad (5\text{-}85)$$

where K is a specified constant that is added to all terms of $|H(u,v)|^2$. The following examples illustrate the use of this expression.

Minimum Mean Square Error (Wiener) Filtering

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Left: degradated image

Middle: inverse filtering

Right: Wiener filtering